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An effective-medium model for high-impedance surfaces

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ABSTRACT We show that high-impedance surfaces with complex microstructures can be well described by a double-layer effective-medium model consisting of an anisotropic meta-material layer (with a dispersive permeability μ) put on top of a metal sheet. We predict that a complete surface wave (SW) gap exists in such a system when the condition $0 < \mu < 1$ is fulfilled, and we verify this prediction by finite-difference-time-domain (FDTD) simulations on realistic structures. We argue that opening a SW gap in the in-phase reflection regime requires an additional mechanism for such kinds of systems, and demonstrate this argument via FDTD simulations on two commonly adopted structures.

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High-impedance surfaces (HIS) have attracted considerable attention in recent years [1–10]. Such structures usually consist of planar periodic metallic arrays printed on a metal-backed dielectric slab either including vertical vias [1–4] or not [5–9]. This family of systems exhibits two interesting electromagnetic (EM) properties. First, the phase of the EM wave reflected by the HIS varies continuously from 180° to −180° versus frequency, indicating that the system must reflect the EM wave in-phase at a specific frequency. Second, the surface waves (SW) with both polarizations are completely suppressed in a particular frequency window. Owing to these two fascinating properties, the HIS have found several interesting applications in practice, such as being the ground planes for antennas [1, 5, 6] or working as the side walls of sub-wavelength cavities [11, 12].

Previous theoretical understandings of such systems were mostly based on brute-force numerical computations, such as finite-difference-time-domain (FDTD) simulations [2, 5, 6] or the finite-element method [1]. Such methods can yield highly reliable results that can be directly compared with experiments. However, due to the complexity and the diversity of the structures, numerical computations are usually time consuming and the obtained results are not easily transplanted between different structures. In addition, from such full-wave calculations, it is not easy to identify which element is crucial to induce the in-phase reflectivity and which is to open the SW gap, and to understand the relationship, if any, between these two characteristics. Equivalent circuit models have been widely used to describe the characteristics of HIS [1, 13]. However, these methods are strictly valid only for simple structures. In addition, such models can usually only account for the reflective properties (i.e. the in-phase reflection) of the HIS, but face difficulties in understanding the rich SW characteristics in such composite systems [1].

In this letter, we show that all these high-impedance surfaces, no matter how complex they appear, can be modeled by a double-layer system consisting of a homogeneous anisotropic meta-material layer (with a dispersive permeability matrix $\overleftrightarrow{\mu}$) put on top of a metal sheet (see Fig. 1d). This simplified effective-medium model cannot only account for the reflective properties, but also the SW properties, of the realistic HIS. In particular, the model predicts a complete SW gap as the condition $0 < \mu_{||} < 1$ is fulfilled, which is subsequently verified by FDTD simulations on realistic structures. In addition, we find that those vertical vias in realistic HIS play a crucial role in opening the SW gap in the in-phase reflecting regime, through the Bragg mechanism.

We illustrate our ideas through considering a realistic HIS consisting of an array of metallic Jerusalem crosses on a metal-backed slab (without vias) [4], with a top view of the geometry shown in Fig. 1a, and a side view shown in Fig. 1c. The structural parameters are taken as $a = 7$ mm,

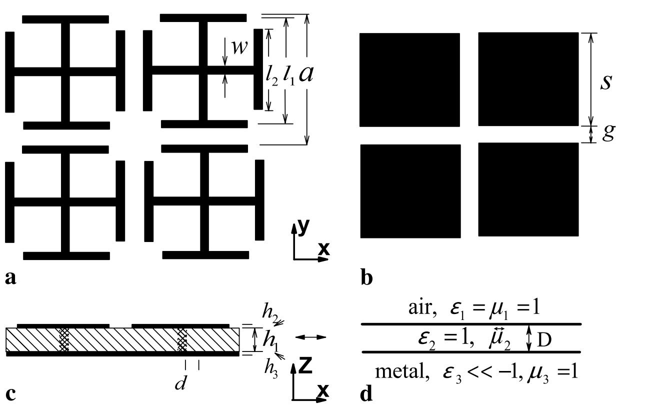

FIGURE 1 Geometries of the patterns studied in this paper: (**a**) Jerusalem crosses, (**b**) square patches. (**c**) Schematic side view of the high-impedance structures (with or without vertical vias). (**d**) Geometry of our double-layer effective-medium model for the high-impedance surfaces

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$w = h_2 = 0.2$ mm, $h_1 = 0.8$ mm, $l_1 = 5$ mm, $l_2 = 4$ mm and the dielectric constant of the inner slab is taken as $\varepsilon = 4.0$. We have calculated its reflection properties through FDTD simulations [14, 15], and the calculated reflection phase (under normal incidence) was plotted as a function of frequency in Fig. 2a. The reflection phase is shown to change continuously from 180° to −180° as frequency increases, and is exactly zero at $f = 7.95$ GHz and 16.65 GHz. From the FDTD simulations, we find that each metallic cross forms a LC circuit with the back metallic sheet, and the induced electric current on each cross is just opposite to that flowing on the metallic sheet. Therefore, there is a considerable H field induced inside the dielectric layer sandwiched between the cross layer and the metallic sheet. This observation motivates us to construct the double-layer effective-medium model as shown in Fig. 1d. Obviously, the thickness of the meta-material layer should be $D = h_1 + h_2$. Geometry restricts that $\mu^{zz} \equiv 1$, since no mechanism is available here to induce an $\boldsymbol{H}$ field along the z direction. Taking the permeability matrix elements of the magnetic material as [16]

$$\mu_2^{xx} = \mu_2^{yy} = \mu_2^{||} = 1 + \frac{19}{7.95^2 - f^2} + \frac{70}{16.65^2 - f^2}, \quad \mu_2^{zz} = 1,$$

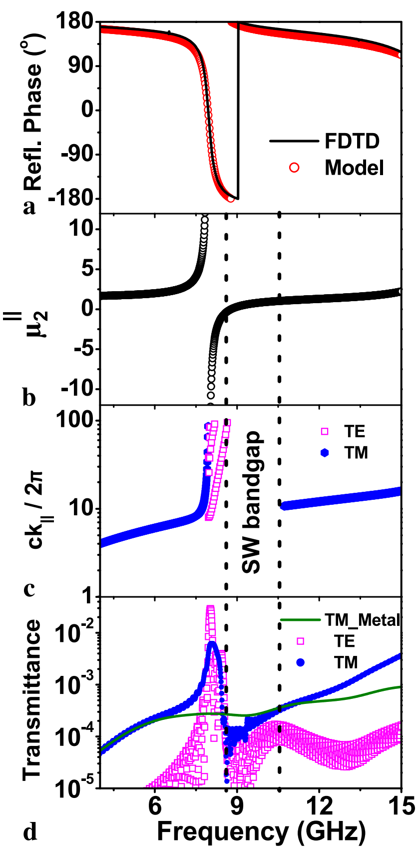

FIGURE 2 (**a**) Reflection phase spectra calculated by the FDTD simulation for the realistic high-impedance surface (*solid line*) and by the double-layer effective-medium model (*open circles*). (**b**) The value of permeability adopted in the effective-medium model. (**c**) Surface wave spectra calculated based on the double-layer effective-medium model. (**d**) FDTD simulated surface wave transmission spectra for the high-impedance surface, referenced by the TM mode surface wave transmission on a flat metal sheet

we employed the transfer-matrix method to study the reflection phase spectrum of the double-layer model. Open circles in Fig. 2a are the results calculated based on such a model. Excellent agreement is found between the model and the FDTD results. We note that the reflection amplitude is exactly 1 in both calculations, since there is a perfect metal sheet in both situations.

We then examine the SW properties of the model system. Following the method described in [10], for the transverse electric (TE) SW mode, we find the following equation to determine its dispersion relation:

$$\left(1 + \frac{\alpha_1 \mu_3}{\alpha_3 \mu_1}\right)\cosh(\alpha_2 d) + \left(\frac{\alpha_1 \mu_2^{||}}{\alpha_2 \mu_1} + \frac{\alpha_2 \mu_3}{\alpha_3 \mu_2^{||}}\right)\sinh(\alpha_2 d) = 0\,, \tag{1}$$

where

$$\alpha_2 = \sqrt{\frac{\mu_2^{||}}{\mu_2^{zz}}\left[k_{||}^2 - (\omega/c)^2 \varepsilon_2 \mu_2^{zz}\right]}$$

and

$$\alpha_l = \sqrt{k_{||}^2 - (\omega/c)^2 \varepsilon_l \mu_l}\,, \quad l = 1, 3\,,$$

in which $k_{||}$ is the parallel component of the $\boldsymbol{k}$ vector. The transverse magnetic (TM) mode satisfies a similar equation with μ_l changed by the corresponding ε_l [10]. The calculated SW dispersion relation of the model system is shown in Fig. 2c. Compared with the value of $\mu_2^{||}$ depicted in Fig. 2b, we understand that the TM mode below the resonance frequency (7.95 GHz) is apparently contributed by the metal surface since the meta-material layer is now transparent ($\mu_2^{||} > 0$), so that the EM wave can 'see' the metal surface. On the other hand, the TE surface mode located in the middle must be dominated by the air/meta-material interface, since $\mu_2^{||}$ of the meta-material layer is now negative and thus the air/meta-material interface can support a TE surface mode. The most interesting observation is that a complete SW band gap appears in the frequency window when the condition $0 < \mu_2^{||} < 1$ is satisfied. The physics can be understood from the following argument. We note that a TM-SW mode bounded at the meta-material/metal interface possesses a dispersion relation $k_{||} = \sqrt{\mu_2^{||}}\,(\omega/c)$ in the limit of $\varepsilon_3 \to -\infty$ (in the case of a good metal). However, since $k_{||} = \sqrt{\mu_2^{||}}\,(\omega/c) < \omega/c$ as $\mu_2^{||} < 1$, such a mode cannot be bounded in the air region (requiring $k_{||} > \omega/c$) as a surface wave. Considering the fact that a TE-SW mode requires a negative $\mu_2^{||}$, we conclude immediately that neither a TE nor a TM mode exists when the condition $0 < \mu_2^{||} < 1$ is satisfied.

We have employed FDTD simulations [14, 15] to study the SW transmission spectra on the realistic HIS with crosses. For the TM (TE) mode, we placed a monopole antenna perpendicular (parallel) to the HIS as a signal source, and measured the transmission signal through a loaded reference plane placed far away [17]. The FDTD calculated SW transmission spectra on the realistic HIS are shown in Fig. 2d, referenced by the transmission spectrum of the TM-SW mode on a flat

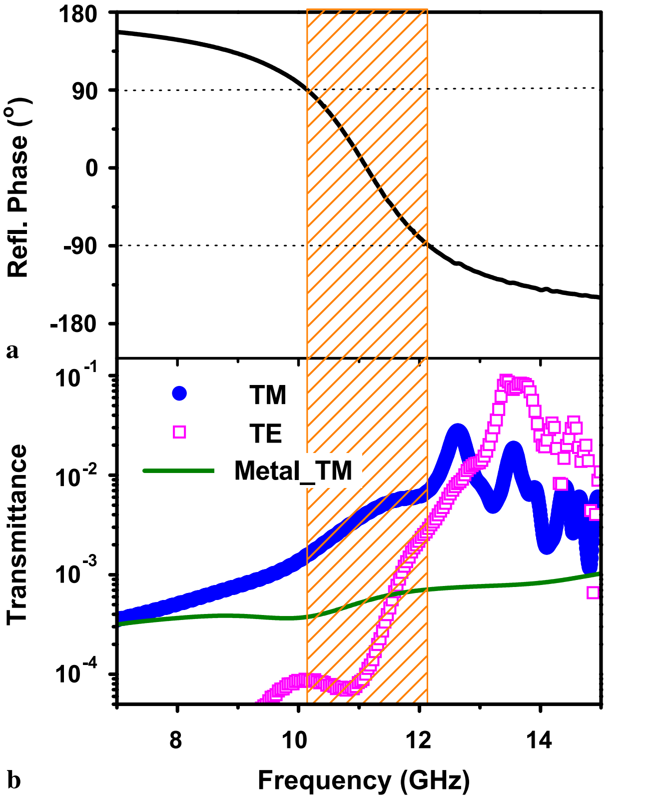

FIGURE 3 FDTD simulated (**a**) reflection phase spectrum and (**b**) surface wave transmission spectra of the high-impedance surface with unit cell being the metallic square patches (without vias). The in-phase reflection regime is indicated as the *shadowed area*

metal surface of the same size. The simulated transmission spectra on the realistic sample are found to coincide with the SW dispersion relation calculated based on the double-layer effective-medium model. In particular, both TE- and TM-mode surface waves are strongly suppressed in the band gap frequency regime predicted by the model calculation. In viewing the good agreements shown in Fig. 2, we conclude that we have established a valid effective-medium model for such composite systems.

For this effective-medium model, we find that the complete SW gap (where $0 < \mu_2^{||} < 1$) does not overlap with the in-phase reflection frequency regime (where $\mu_2^{||} \to \pm\infty$). This is understandable since we expect a TM mode as $\mu_2^{||} \to +\infty$ and a TE mode as $\mu_2^{||} \to -\infty$ and, therefore, there does not exist a regime where both modes are forbidden. The transmission spectra on a realistic HIS shown in Fig. 2d unambiguously support this conclusion. We find that this conclusion is valid for all such HIS (without vertical vias). To demonstrate this point, we consider a different HIS with unit cell as a square metallic patch (see Fig. 1b). The structural parameters of the HIS are fixed as $s = 4.5$ mm, $g = 0.5$ mm $h_1 = h_3 = 0.2$ mm and $h_2 = 0.8$ mm. We have studied its reflective and SW properties via FDTD simulations [14, 15], and have depicted the calculated reflection phase spectrum in Fig. 3a and the SW transmission spectra in Fig. 3b, respectively. In consistency with the predictions given by our double-layer model, the system supports TM (TE) mode SW below (above) the magnetic resonance frequency, and there does not exist a complete band gap in the in-phase reflection regime.

It is thus clear that opening a SW gap in the in-phase reflection regime requires an additional mechanism for such kinds of systems. We find that Bragg scattering is one of such mechanisms, and the vertical vias in many such systems [1–4] play such a crucial role. Shown in Fig. 4 are the FDTD calculated reflection phase and SW transmission spectra for the two HIS studied in this letter, both with metallic vertical vias (radius $d = 1.5$ mm, see Fig. 1c). The comparisons between the SW transmission spectra (Fig. 4c and d) and the reflection phase

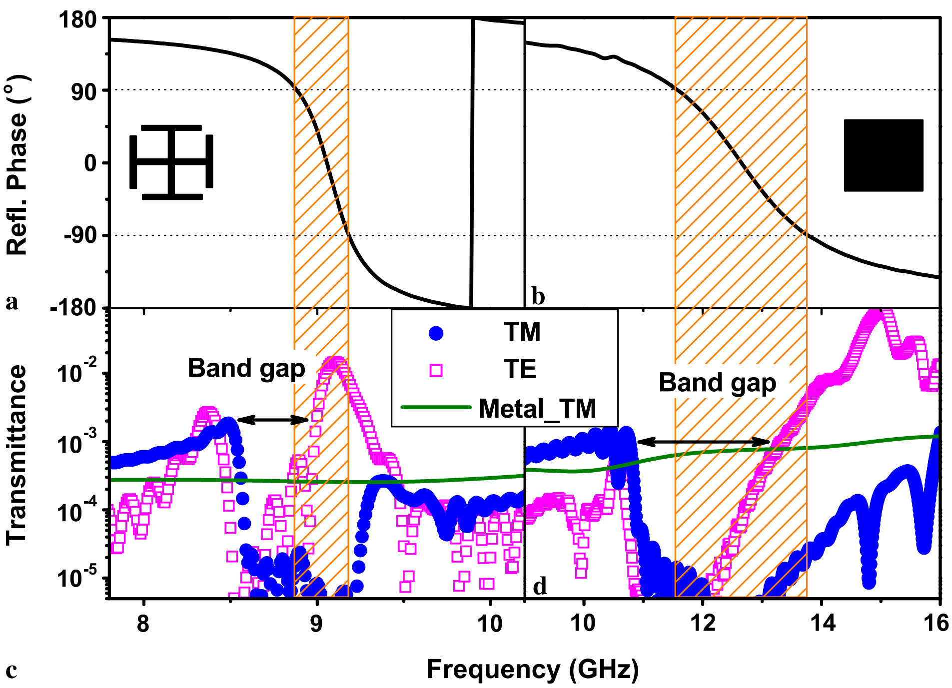

FIGURE 4 FDTD simulated (**a**) reflection phase spectrum and (**c**) surface wave transmission spectra for the high-impedance surface with unit cell as a cross. FDTD simulated (**b**) reflection phase spectrum and (**d**) surface wave transmission spectra for the high-impedance surface with unit cell as a square. Both structures have connecting vias with diameter $d = 1.5$ mm

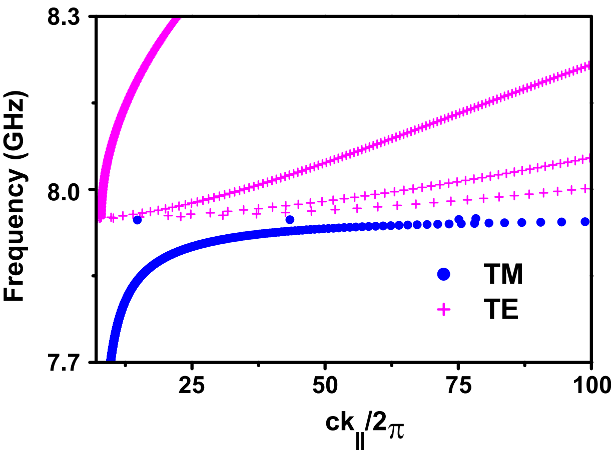

FIGURE 5 An expanded view of the surface wave dispersion relation for the double-layer effective-medium model near the magnetic resonance

spectra (Fig. 4a and b) indicate that complete SW gaps indeed open in the in-phase reflection regimes, for both types of structure. The role of the vertical vias is now clear. Besides shifting the resonance frequency slightly, the dominant effect of those vertical vias is to create strong Bragg scatterings so as to help open a complete band gap [18].

We find from the transmission spectra that the TM mode is relatively easy to be 'killed' by the vertical vias while the TE mode is relatively difficult. As a result, the center frequency of the SW gap is down shifted with respect to the in-phase reflecting frequency, which was also observed in [5, 9]. The physics is also easy to understand from our effective-medium model. Shown in Fig. 5 is an enlarged view of the SW spectra near the resonance frequency of the double-layer model for the cross structure. While the dispersion of TM polarization exhibits a single-mode character, the dispersion relation of TE polarization exhibits a multi-mode character, which is caused by the anisotropic nature of the magnetic meta-material. We suspect that this fact possibly accounts for the observation discussed above, since it is naturally more difficult to suppress multiple modes simultaneously than to suppress a single mode, through the Bragg scatterings.

In conclusion, we have established an effective-medium model for high-impedance surfaces, and demonstrated its validity to describe both the reflective and surface wave properties of the realistic structures. We argue that the appearance of a SW band gap in the in-phase reflection regime requires an additional mechanism, and the metallic vertical vias in such structures play crucial roles to open SW gaps through Bragg scatterings.

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15 Simulations were performed using the package CONCERTO 4.0, developed by Vector Fields Ltd, England (2004). In our simulations, a basic cell sized 0.5 mm × 0.5 mm × 1 mm is adopted to discretize the space. Finer sub-meshes were adopted in space regions where strong inhomogeneity exists
16 For planar patterns without x–y symmetry like that in [6], we should set $\mu_2^{xx} \neq \mu_2^{yy}$
17 In both TE and TM excitation cases, we find that the in-plane electric dipoles are induced on the metallic patterns, which further couple to the metal sheet to form magnetic responses
18 We find that the Bragg scatterings introduced by the periodic planar metallic patterns are not strong enough to open the gap